

Gapless color-flavor locked phase in quark and hybrid stars

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We study the effects of the gapless color-flavor locked (gCFL) phase on the equation of state of strongly interacting matter in the range of baryonic chemical potential involved in a compact star. We analyze the possibility of a phase transition from hadronic matter to gCFL quark matter and we discuss, for different values of the strange quark mass and diquark coupling strength, the existence of a gCFL phase in quark or hybrid stars. The mass-radius relation and the structure of compact stars containing the gCFL phase are shown and the physical relevance of this superconducting phase inside a stellar object is also discussed.

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I. INTRODUCTION

Recent studies on the QCD phase diagram at finite densities and temperatures have revealed the existence of a rich structure of the phase diagram with several possible types of color superconducting phases [1, 2]. These results are very interesting in the study of the structure and formation of compact stellar objects in which the central density may reach values up to ten times the nuclear matter saturation density and therefore deconfinement of quarks may take place. Similar conditions may also be reached in future heavy ion colliders, as at GSI, where it will be possible to study the transition from hadronic matter to quark gluon plasma, and the possible low-temperature transition from hadronic matter to color superconducting quark matter.

The structure of the QCD phase diagram at high densities and vanishing temperature (the conditions in a compact star) depends strongly on the value of the strange quark mass m_s . For the two extreme cases, vanishing m_s and very large values of m_s , it is widely accepted that the three-flavor color-flavor locked (CFL) phase and the two-flavor color superconducting phase (2SC) are, respectively, the most favored phases [1, 2]. At intermediate values of m_s , it is in general difficult to involve strange quarks in BCS pairing due to their Fermi momentum, which is lower than that of up and down quarks, therefore the CFL phase can form only if the CFL superconducting gap, Δ_{CFL} , is large enough [3]. Recently, it has been shown that the CFL phase can form only if the ratio $m_s^2/\mu \lesssim 2\Delta_{CFL}$ [4]. At larger values of m_s^2/μ , but not too large values of m_s , the most energetically favored phase is the so-called gapless CFL (gCFL) phase instead of the 2SC phase or unpaired quark matter (UQM) [5, 6, 7]. The gCFL phase has the same symmetries as the CFL phase but there are two gapless quark modes and a nonzero electron density. The existence of gapless degrees of freedom makes the gCFL phase a conductor, at variance from the CFL phase, and it is expected to have very different transport properties with respect to the CFL phase. The relevance of the gCFL phase in

the equation of state of strongly interacting matter at low temperature is actually controversial and depends on the value of the diquark coupling, on the strange quark mass and on the baryonic chemical potential. In particular, it has very recently been shown that on taking into account dynamical chiral symmetry breaking within the NJL model, the 2SC phase would be the favored phase at low densities if large values of the diquark coupling were used [8, 9, 10]. Although there are uncertainties in the region of the QCD phase diagram in which the gCFL phase occurs, as outlined in Refs.[4, 5, 6], the gCFL phase may have a relevant role in compact stars because of the wide range of baryonic chemical potential involved in these stellar objects. A first study of the effect of the presence of the gCFL phase in a compact star was presented in Ref.[11]. In that paper the authors compute the specific heat and neutrino emissivity of the gCFL phase claiming that if the gCFL phase forms in an old compact star, it should deeply affect its cooling. These results are obtained assuming that the star is composed of a nuclear matter crust and a gCFL core at a fixed value of the quark chemical potential ($\mu = 500$ MeV), corresponding approximately to ten times the nuclear baryon density, the density reached in the core of a compact star. Actually, a detailed investigation on the relevance of the gCFL phase on the structure and formation of a compact star is still lacking in literature.

The main goal of this paper is to study the possible existence of the gCFL phase in the large range of the baryonic chemical potential involved in compact stars. We will compute, for different values of the strange quark mass and diquark coupling strength, the equation of state (EOS) of the gCFL phase considering also the possibility of a phase transition from hadronic matter to gCFL quark matter. The resulting EOSs will then be used to obtain the structure of both quark and hybrid stars. This paper is organized as follows: in Sections II and III, we study the relativistic hadronic EOS and the three quark flavors EOS (gCFL-CFL), respectively; in Sec. IV we discuss the possibility of phase transitions from hadronic matter to quark matter; in Sec.V we compute the mass-radius relations of quark and hybrid stars; the conclu-

sions are reported in Sec.VI.

II. EQUATION OF STATE OF HADRONIC MATTER

Concerning the hadronic phase, we use a relativistic self-consistent theory of nuclear matter in which nucleons interact through the nuclear force mediated by the exchange of virtual isoscalar and isovector mesons (σ, ω, ρ) [12]. At $T = 0$, in the mean field approximation, the thermodynamic potential Ω per unit volume can be written as

$$\Omega = -\frac{1}{3\pi^2} \sum_B \int_0^{k_{FB}} dk \frac{k^4}{E_B^*(k)} + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} a \sigma^3 + \frac{1}{4} b \sigma^4 - \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{2} m_\rho^2 \rho^2, \quad (1)$$

where the \sum_B runs over the eight baryon species, $E_B^*(k) = \sqrt{k^2 + M_B^{*2}}$ and the baryon effective masses are $M_B^* = M_B - g_\sigma \sigma$. The effective chemical potentials ν_B are given in terms of the thermodynamic chemical potentials μ_B and of the vector meson fields as follows

$$\nu_B = \mu_B - g_\omega \omega - t_{3B} g_\rho \rho, \quad (2)$$

where t_{3B} is the isospin 3-component for baryon B and the relation to the Fermi momentum k_{FB} is provided by $\nu_B = \sqrt{k_{FB}^2 + M_B^{*2}}$. The isoscalar and isovector meson fields (σ, ω and ρ) are obtained as a solution of the field equations in mean field approximation and the related couplings (g_σ, g_ω and g_ρ) are the parameters of the model [12, 13, 14].

In Fig. 1 we display the relative concentrations of the various particle species $Y_i = \rho_i / \rho_B$ as a function of baryonic density ρ_B by imposing charge neutrality and β -equilibrium for the GM3 parameter set [12].

III. EQUATION OF STATE OF THE GAPLESS CFL PHASE

To compute the EOS of the gCFL phase we adopt the NJL-like formalism of Refs.[4, 5, 6] in which the thermodynamic potential per unit volume can be written as

$$\Omega = -\frac{1}{4\pi^2} \int dp p^2 \sum_j |\epsilon_j(p)| \rho_j(p) + \frac{1}{G} (\Delta_1^2 + \Delta_2^2 + \Delta_3^2) - \frac{\mu_e^4}{12\pi^2}, \quad (3)$$

where $\Delta_1, \Delta_2, \Delta_3$ are the superconducting gaps characterizing the gCFL phase (which reduce to a single gap in the CFL phase), G is the strength of the diquark coupling, $\epsilon_j(p)$ are the dispersion relations of quarks as in Ref. [5] and μ_e is the electron chemical potential. Following the approximations used in Refs.[4, 5], the effect

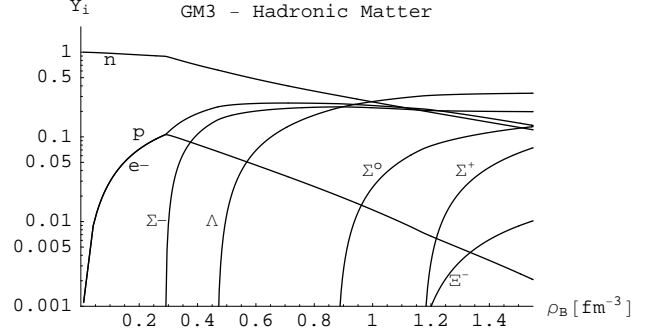


FIG. 1: Particle fractions Y_i of neutral and β -stable hadronic matter as a function of baryonic density ρ_B for the GM3 hadronic equation of state [12]

of m_s is introduced as a shift $-m_s^2/2\mu$ in the chemical potential for the strange quarks and the contributions of antiparticles is neglected. The first approximation is applicable for values of m_s smaller than the chemical potential and therefore we will use in this paper typical values of m_s of 150 – 200 MeV. Concerning the antiparticles, as already remarked in Ref. [5], neglecting their contribution can lead to incorrect values of the thermodynamic variables. In that paper, this problem does not play a relevant role because the free energy differences relative to UQM are presented. Here we are going to investigate the structure of a compact star and we need to compute the variation of the thermodynamic potential as a function of the chemical potential. To this end, following [15, 16], we have introduced in Eq.(3) the quasiparticle probabilities

$$\rho_j(p) = \frac{1}{2} \left(1 - \frac{\tilde{\epsilon}_j(p)}{\epsilon_j(p)} \right), \quad (4)$$

where $\tilde{\epsilon}_j(p)$ are the dispersion relations with vanishing gaps. To assure the convergence of the integral in Eq.(3), a form factor $f = (\Lambda^2/(p^2 + \Lambda^2))^2$, which multiplies the gaps, is introduced in the dispersion relations $\epsilon_j(p)$. The form factor was fixed to mimic the effects of the asymptotic freedom of QCD [16] and the parameter Λ was fixed at a value of 800 MeV. For the CFL phase, this procedure leads to results in agreement with previous calculations performed using the simplified model of Refs. [17, 18].

To describe the matter of a compact star, the conditions of chemical equilibrium between quarks, charge neutrality and color neutrality must be imposed. The chemical equilibrium conditions (which also include β -stability) allow the expression of each quark chemical potential μ_{cf} (c and f are the indexes of color and flavor, respectively) as functions of quark (baryonic) chemical potential μ , electron chemical potential μ_e and the two chemical potentials, μ_3 and μ_8 , associated to the $U(1) \times U(1)$ subgroup of the color gauge group (see Ref. [5] for details). The color and electric charge neu-

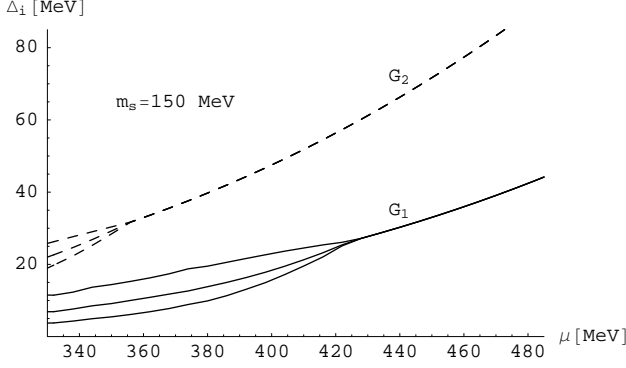


FIG. 2: Gap parameters as a function of the quark chemical potential for two different values of diquark coupling G_1 and G_2 and for a fixed value of strange quark mass $m_s = 150$ MeV. The larger the value of G , the larger the window of the chemical potential in which CFL phase occurs.

trality are imposed by the following three equations:

$$\frac{\partial \Omega}{\partial \mu_3} = 0, \quad \frac{\partial \Omega}{\partial \mu_8} = 0, \quad \frac{\partial \Omega}{\partial \mu_e} = 0. \quad (5)$$

Moreover, the thermodynamic potential must be minimized with respect to the gap parameters and therefore we have to impose the three additional conditions

$$\frac{\partial \Omega}{\partial \Delta_1} = 0, \quad \frac{\partial \Omega}{\partial \Delta_2} = 0, \quad \frac{\partial \Omega}{\partial \Delta_3} = 0. \quad (6)$$

The above equations allow us to compute the thermodynamic potential and all the thermodynamic variables as a function of the quark chemical potential only. On solving these equations, we obtained results in agreement with the ones shown in Ref.[5] concerning the dispersion relations, the gaps and the chemical potentials as a function of m_s/μ .

In Figs. 2 and 3 the gap parameters are displayed as functions of quark chemical potential μ for two different values of m_s . Here we have used two values of the diquark coupling G_1 and G_2 corresponding, respectively, to values of $\Delta_{CFL} \sim 40$ and $\Delta_{CFL} \sim 100$ MeV at $\mu = 500$ MeV and $m_s = 150$ MeV. It is interesting to observe that the window in which the gCFL phase appears depends noticeably on the value of the diquark coupling and, in particular, it increases with m_s and decreases with G (see Figs. 2 and 3). This confirms the general argument for which the transition from gCFL to CFL occurs when $m_s^2/\mu \simeq 2\Delta_{CFL}$.

In Figs. 4 and 5 we compare the EOSs of the (g)CFL and UQM phases. A bag constant ($B^{1/4} = 150$ MeV in Figs. 4 and 5) has been added to the thermodynamic potential to simulate quark confinement, as in the MIT bag model. In Fig. 4, we display the pressure of the gCFL and UQM phases for two choices of coupling G at a fixed value of m_s . Let us observe that the smaller values of the

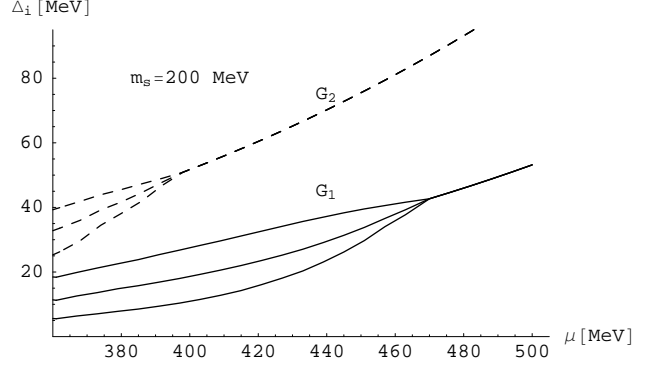


FIG. 3: The same as Fig.2, with $m_s = 200$ MeV.

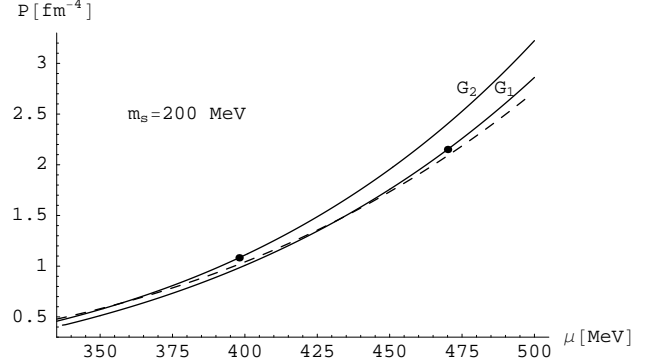


FIG. 4: Pressure as a function of the quark chemical potential for UQM (dashed line) and for the (g)CFL phase (solid lines) for two values of diquark coupling G_1 and G_2 . The dots represent the onset of the gCFL-CFL phase for the two coupling parameters G_1 and G_2 . The bag constant is fixed at a value $B^{1/4} = 150$ MeV.

diquark coupling imply a larger chemical potential window in which gCFL occurs; however, only a part of this window is favored in comparison with UQM. In fact, from Fig. 4 it can be observed that gCFL is more energetically favored than UQM for a window of about $\Delta\mu \simeq 30 - 50$ MeV. It is also to be noted that the CFL phase is always favored at large chemical potentials. A similar behavior was also found in Ref. [9] on taking into account the dynamic generation of quark masses. Calculation of the energy per baryon as a function of the baryon density is shown in Fig. 5. It is also evident in this figure that gCFL is energetically favored with respect to UQM in a strict region of baryon density. The EOSs displayed in Fig. 5 satisfy the Bodmer-Witten hypothesis [19, 20, 21] which states that the true and absolute ground state of the strong interaction is quark matter. As we will see in Sec. V, this will lead to stellar objects entirely occupied by quark matter.

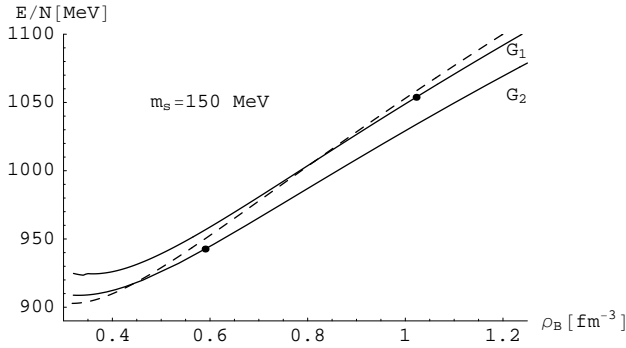


FIG. 5: Energy per baryon displayed as a function of baryon density for fixed values of the bag constant $B^{1/4} = 150$ MeV and the strange quark mass. The dashed line corresponds to UQM and the solid lines correspond to (g)CFL EOSs for the two choices of diquark coupling G_1 and G_2 . The dots represent the onsets of the gCFL-CFL phase.

IV. PHASE TRANSITION FROM HADRONIC MATTER TO THE GAPLESS CFL PHASE

The region of the QCD phase diagram in which the transition from hadronic matter to quark matter should occur, is little known. In particular, it is not yet known if there is a direct transition from hadronic matter to the ground state of QCD, i.e. the CFL phase, or if there is an intermediate density window in which another quark phase may appear. In several papers [18, 22, 23, 24], it was assumed that this transition is direct by using an approximate equation of state valid for large values of the gap parameters. However, as already remarked, the gCFL phase may be a valid candidate in connecting hadronic matter to the CFL phase at intermediate densities and it is very important to see if the first order phase transition between hadronic matter and gCFL possibly occurs via an intermediate density window of mixed phase.

The EOS appropriate to the description of a compact star has to satisfy β -stability conditions. This implies the existence of two conserved charges, the baryonic charge and the electric charge. When the Gibbs conditions are applied in presence of more than a single conserved charge, the technique developed by Glendenning [13] has to be adopted and the equivalence of the baryon and the charge chemical potentials in the two phases must be imposed. Moreover, the electric neutrality is required as a global condition [37]. The corresponding equation reads:

$$(1 - \chi)\rho_c^H + \chi\rho_c^{gCFL} - \rho_e = 0, \quad (7)$$

where χ is the volume fraction of the quark phase, ρ_c^H is the charge density of the hadronic phase given by protons and charged hyperons if present, ρ_c^{gCFL} is given by the density of the gapless bu quarks (see Ref. [5] for details),

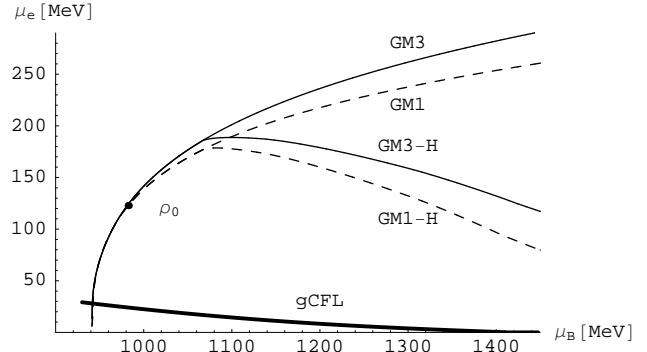


FIG. 6: Electrons chemical potential as a function of the baryon chemical potential for the pure neutral hadronic and gCFL phases. Solid and dashed lines correspond to the nuclear (np) and hadronic (npH) matter in models GM1 and GM3 of Ref.[12]. The thick solid line corresponds to the gCFL phase with $m_s = 200$ MeV and for diquark coupling G_1 . The dot labeled with ρ_0 corresponds to nuclear matter saturation density.

ρ_e is the density of electrons.

In computing the first critical density ($\chi = 0$), Eq.(7) coincides with the charge neutrality equation for the pure hadronic matter. Since the transition to a mixed phase of hadronic-quark matter eventually takes place at densities larger than nuclear matter density ρ_0 , the electron chemical potential, as can be seen in Fig.6, is larger than $\mu_e \sim 100$ MeV. It turns out that for such a high value of μ_e , the resulting splitting of quarks' Fermi momenta is too large for the given diquark coupling strength to enable gCFL diquark pairing. Moreover, in the mixed phase UQM may contribute with a total negative charge density which neutralizes the positive charge of protons and the chemical potential of electrons decreases to few MeVs. Since the gCFL phase contributes with a positive charge density given by the gapless bu quarks, the chemical potential of electrons must therefore have large values to neutralize the positive charges of hadron and quark phases. Let us remark that at this stage the formation of meson condensates [25] has not been considered in the gCFL phase. The formation of Goldstone bosons may influence noticeably the mixed phase at intermediate chemical potentials. Unfortunately, no investigations on this possibility are present in literature. Moreover, if the 2SC phase appears at low densities (i.e. for large values of m_s and the diquark coupling parameters) the creation of the mixed phase may be favored. The relevance of these possibilities lies beyond the scope of this paper and will be investigated in future works.

If Gibbs conditions cannot be realized, the phase transition between hadronic matter and gCFL may occur with a discontinuity in the baryon density by means of the Maxwell construction. This would be the case even if a large surface tension between the hadronic and quark

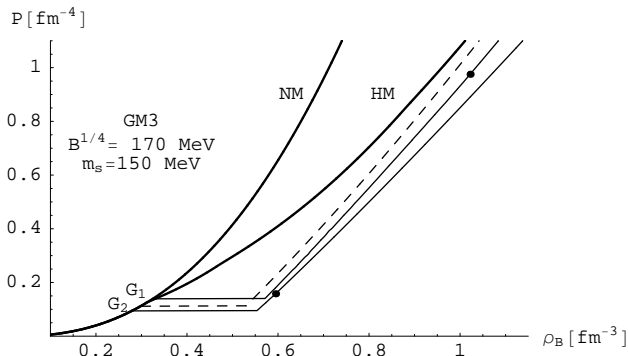


FIG. 7: Pressure as a function of baryon density for different choices of parameters G . The thick solid lines correspond to the nuclear and hadronic matter GM3 EOSs (NM and HM). The dashed line represents the case in which a phase transition from hadronic matter to UQM is considered using the Maxwell construction. The thin solid lines are related to the phase transition with the gCFL phase for two choices of diquark pairing G . The dots on the solid lines correspond to phase transition from gCFL to CFL phases (G_1 and G_2 have the same values as in Fig. 2 and 3).

phases exists. In Fig. 7 the EOSs obtained using the Maxwell construction are displayed. Concerning the effect of the diquark coupling, the larger the value of G the smaller the values of the two critical densities. Notice that the density window in which the gCFL phase appears (the dots on the solid curves indicate the onset of the gCFL-CFL phase) decreases sharply with increases in the value of G . The curve corresponding to the transition from hadronic matter to UQM is shown in the same figure using the Maxwell construction (dashed line). As we can see from Fig. 7, if the transition from hadronic matter to quark matter occurs with discontinuity in the baryon density, the EOS of the gCFL phase labeled with G_2 is favored with respect to the UQM EOS. If instead we take into account the possibility of the formation of a mixed phase, the transition from hadronic matter to quark matter would occur via a hadron-UQM phase transition and then at larger densities the gCFL and CFL phases may appear. This scenario is shown in Fig. 8 from which we can observe that for large diquark coupling (G_2) the transition from the mixed phase to the (g)CFL phase occurs above the onset of the gCFL-CFL phase and for small diquark coupling (G_1) a window of the gCFL phase is instead present. Concerning the hyperons, within this choice of parameters the transition from hadronic matter to gCFL quark matter occurs before reaching the threshold of the formation of hyperons. If we use larger values of the bag constant the first critical density may be larger than the threshold of the formation of hyperons. In that case, however, the phase transition would involve the CFL phase directly.

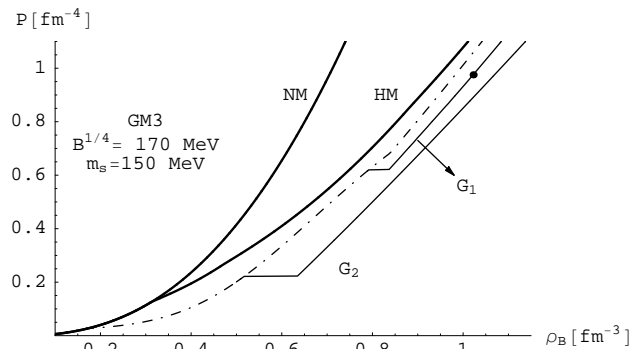


FIG. 8: Pressure as a function of baryon density for the scenario in which a first phase transition from hadronic matter to quark matter occurs via a mixed phase and then a second phase transition (here computed using the Maxwell construction) occurs from this mixed phase to the gCFL phase. The dot-dashed line is related to the hadron-UQM mixed phase. The thin solid lines represent the phase transitions from the mixed phase to the gCFL phase for the two diquark couplings. Notice that in the case of G_1 a gCFL phase window is still present.

V. MASS-RADIUS RELATION OF COMPACT STARS

The EOSs analyzed in the last sections for the gCFL phase can be used to compute the structure of quark and hybrid stars. The EOS is an input function needed to solve the Tolman-Volkoff-Oppenheimer system of equations. The mass-radius relations for quark stars and hybrid stars are displayed in Fig. 9. Concerning quark stars, the effect of the presence of the gCFL phase is not very appreciable and leads to a small reduction of the radius of the star (see solid and dashed lines in Fig. 9 labeled with gCFL-QS and UQM-QS) and it can hardly be considered as a signature for the presence of the gCFL phase in quark stars. Concerning hybrid stars, the way the transition from hadronic matter to gCFL occurs, i.e. with a density discontinuity, is also reflected also in the mass-radius relations of gCFL hybrid stars. Stars in which a very small fraction of the gCFL phase is present in the core are in fact unstable, as can be seen from the initial part of the line corresponding to the branch of hybrid stars (see dotted lines labeled G_1 and G_2 in Fig. 9). On increasing the fraction of quark matter in the volume of the stars, the radii decrease and stable configurations are obtained (see solid lines in Fig. 9). As suggested in Ref. [26], this effect is explained by considering that the adiabatic index of the EOS near the two critical densities is vanishing (using the Maxwell construction) and therefore the pressure is too weak a function of energy density to sustain stability. Even when using Gibbs conditions, similar instabilities are obtained within particular choices of the parameters as shown in Ref. [27].

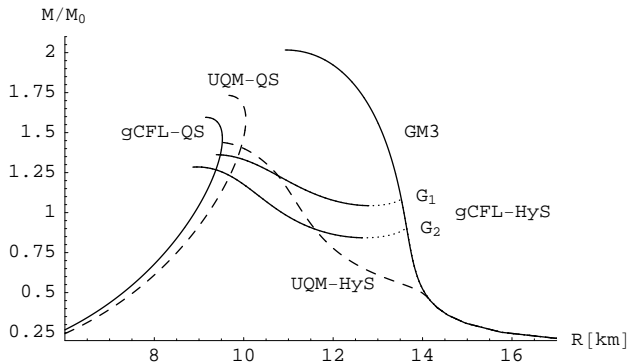


FIG. 9: Mass radius relations of quark stars (QS), hybrid stars (HyS) and hadronic stars (GM3) are displayed for different EOSs. The initial part of the branch of gCFL hybrid stars corresponds to unstable star configurations (dotted lines). As the fraction of gCFL increases in the core of hybrid stars the corresponding stars are stable (solid lines). The EOSs used to compute the mass-radius relations are the same of those in Figs. 5 and 7. In particular, concerning quark stars, the curve labeled gCFL-QS corresponds to the gCFL EOS with diquark coupling G_2 .

It is also important to study the composition of quark and hybrid stars in which the gCFL takes place. In Figs. 10 and 11, the baryon density profile inside quark stars and hybrid stars are shown, respectively. In quark stars, the baryon density does not in general vary very much with the radius of the star. Therefore, it is possible that almost all the star may actually be composed of the gCFL phase (see Fig. 10 for the $1.2M_\odot$ star) with a small core of the CFL phase present. For more massive quark stars, the fraction of the volume occupied by the CFL phase increases (see Fig. 10 for the $1.4M_\odot$ star). In both cases we expect the transport properties of the matter of the star to be determined by the gCFL phase because the CFL phase is essentially passive with all quarks gapped. In general, the presence of a color superconducting phase has relevant effects on the cooling of the stars as shown in Refs. [11, 28, 29]. Moreover, the presence of the gCFL phase can also be very important in the study of r-mode instability which imposes rather severe limits on the highest rotation frequency of pulsars [30, 31]. Indeed, we expect the gapless modes of the gCFL phase to play an important role in the bulk and shear viscosity of the star [32]. For instance, in Ref. [33] it is shown that the existence of pure CFL quark stars is ruled out by the existing data on pulsars because of r-mode instability [38]. A crust of the gCFL phase may help to stabilize the star and therefore quark stars may again be considered as possible stellar objects. Concerning hybrid stars, their structures are shown in Fig. 11 using the EOS of Fig. 8 for diquark coupling G_1 . As in the case of quark stars, the volume occupied by the gCFL phase varies with the mass of the star. In this case a crust of hadronic matter

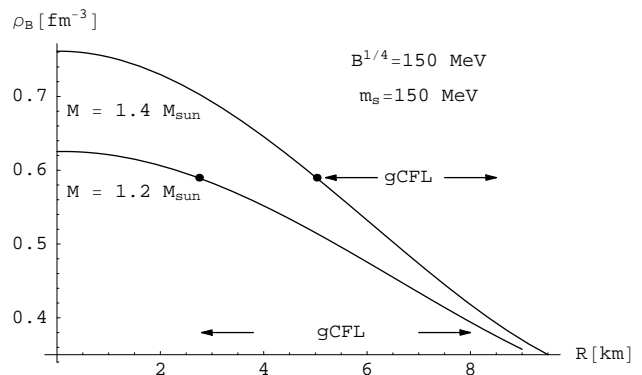


FIG. 10: The baryon density profiles are shown for two quark stars having a mass of $1.4M_\odot$ and $1.2M_\odot$. The region of the stars indicated by the arrows is composed of the gCFL phase. The diquark coupling used in the corresponding EOS is G_2 .

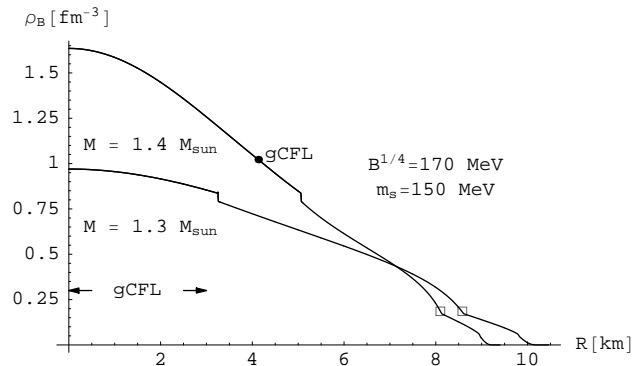


FIG. 11: The baryon density profiles are shown for two hybrid stars having a mass of $1.4M_\odot$ and $1.3M_\odot$. The dot indicates the onset of gCFL-CFL and the squares indicate the beginning of the mixed phase. The diquark coupling used in the corresponding EOS is G_1 .

and a large layer of mixed phase are present. Depending on the mass of the star, a narrow layer of the gCFL phase ($M = 1.4M_\odot$) or a core of the gCFL phase may form ($M = 1.3M_\odot$). As suggested in Ref.[11], the core or the layer of the gCFL phase, which has a high heat capacity, can keep the star warm for a long time, thus affecting the cooling process of the star.

VI. CONCLUSIONS

In this paper we studied the relevance of the presence of the gCFL phase in compact stars. We first computed the equation of state of the gCFL phase and compared it with the unpaired quark matter equation of state. We then investigated the phase transition from hadronic matter to the gCFL phase. We found that the possibility of

the formation of a mixed phase between hadronic matter and the gCFL phase is hindered by the existence in the hadronic phase of a large number of electrons which destroy gCFL pairing in the mixed phase. Therefore, the Maxwell construction was adopted to connect the hadronic to the gCFL phase and we found that the gCFL phase is energetically favored only in a narrow region of the chemical potential. Finally, we studied the effect of the presence of the gCFL phase in both quark and hybrid stars. Although the gCFL phase does not sensibly modify the global properties of stars, masses and radii, we have shown that such a superconducting phase may occupy a wide region inside the star and therefore may play a crucial role in the calculation of the transport properties of the matter of the star with very important physical implications. These results may stimulate more detailed studies on the effects of the gCFL phase in the cooling and in the determination of the bulk and shear viscosity inside a compact star, thus leading to a better understanding of the mechanism acting in the suppression of r-mode instabilities in compact stars. Another interesting phenomenological signature of the presence of the

gCFL phase in compact stars may be provided by analyzing the role of this phase in explosive phenomena such as Supernovae and gamma-ray bursts. It is in fact well known that first order phase transitions in compact stars can release huge amounts of energy [34]. It may therefore be very important to see if the first order phase transition from unpaired quark matter to the gCFL phase can occur during the life of a compact star. A hypothetical phase transition from unpaired quark matter to gCFL quark matter may help to explain the complex time structure of some gamma ray bursts [35] in an astrophysical scenario in which the central engine of gamma ray bursts is given by conversions between different phases of strongly interacting matter.

VII. ACKNOWLEDGMENTS

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 - [37] Color neutrality is instead imposed as a local condition because color is clearly confined in the hadronic phase.
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